

Semi-analytical techniques for investigating thermal non-equilibrium effects in wet steam turbines

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This paper describes a semi-analytical solution of the polydispersed wet steam equations, valid in regions where the nucleation rate is negligible. The solution can be used in conjunction with any conventional turbomachinery calculation procedure to obtain estimates of the magnitude of departures from thermal equilibrium. For example, from an initial estimate of the pressure distribution, it is a simple matter to calculate the distribution of supercooling and wetness fraction, together with the thermodynamic losses incurred by the flow.

The method differs from the usual numerical approach by providing general results which give considerable physical insight. Computational time and effort is also dramatically reduced. The controlling parameters emerge naturally from the analysis, and information concerning the fundamental fluid mechanics of wet steam is revealed. In particular, the analysis demonstrates the role played by the thermal relaxation time and the rate of expansion in controlling the deviation from equilibrium.

The versatility and usefulness of the technique in furnishing results for the turbine designer are demonstrated by a number of applications including one-dimensional nozzle flows and two-dimensional blade-to-blade and hub-to-tip flows. In each case it is shown how the droplet size and expansion rate influence the thermodynamic losses and other flow variables of interest

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Numerical calculation procedures for one-dimensional nucleating steam flows show good agreement with data from condensation shock experiments in converging-diverging nozzles¹⁻³. Despite a sparsity of data in some regimes, predictions of pressure and droplet size distributions can be made over a wide range of conditions with considerable confidence⁴. Attempts to apply similar methods to flows in steam turbines have also met with some success^{5,6}, but so far the turbine designer has shown a reluctance to introduce the extra complications into his design procedures and most turbines are still analysed assuming the steam to remain in equilibrium below the saturation line.

Most published work in the literature of wet steam concerns itself with primary nucleation and droplet growth. From the point of view of the steam turbine designer, however, this is just one of a number of phenomena which are of interest and which affect the performance of the machines. The primary nucleation zone occupies a comparatively small region, usually in one particular blade row, and for the remainder of the expansion the droplet number per unit mass of steam remains approximately constant. Secondary nucleations may occur where the

expansion rate is high, but nevertheless the flow is substantially non-nucleating for most of its passage through the wet stages of the turbine. The analysis of this type of flow presents fewer problems than when the droplet number is changing continually and forms the subject of this article.

The fact that the steam is non-nucleating does not imply, of course, that the two-phase mixture is at equilibrium and the vapour phase always remains at least slightly supercooled due to the inability of the condensation rate to keep pace with the changing state of the gas. In regions of high expansion rate the vapour supercooling may be considerable, but the droplet temperature always remains close to the saturation value. The resulting interphase temperature difference not only provides the driving force for condensation, but is also responsible for an overall entropy increase of the flow. This in turn leads to a reduction in turbine efficiency referred to as the thermodynamic wetness loss. Departures from equilibrium also affect the steam density and velocity distribution which in turn may affect the aerodynamic performance of the turbine.

Although the fog droplets in the main body of the flow are very fine ($0.05 < r < 1.0 \mu\text{m}$) and closely follow the vapour streamlines, a small proportion deposit on the surface of the blades and are stripped from the trailing edges in the form of large coarse water droplets ($r > 20 \mu\text{m}$). These droplets are responsible for the erosion of the moving blades, but their

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thermal interaction with the vapour phase is negligible due to their very small total surface area. Therefore they can be ignored in any theory dealing with the effects of condensation on the flow.

By confining the analysis to non-nucleating steam flows, it is tacitly assumed that the droplet size spectrum is already known by, for example, direct measurement in the turbine using a light scattering technique⁷. Alternatively a number of calculations might be performed over a range of different droplet sizes in order to gauge the importance of the various effects. The problem of estimating the size of droplets formed during the primary nucleation in a turbine is, as yet, unsolved and it is doubtful whether a complete understanding will be forthcoming in the foreseeable future. By sidestepping the difficulty, however, and using the comparatively simple analytical techniques described below, the turbine designer can obtain both physical insight and useful quantitative information.

The paper is divided into two parts. In the first the theory of non-nucleating polydispersed wet steam flows is developed from an analytical standpoint. This is in contrast to the usual numerical approach and has the advantages that the controlling parameters emerge naturally from the analysis and the results are easily generalised. In the second part some applications of the theory to real flows in turbines are described. The list is by no means exhaustive, but gives some examples of how the analysis can be used to estimate the magnitude of departures from thermal equilibrium and the aero-thermodynamic consequences.

Gas dynamic equations

Equations governing the flow of wet steam have been derived elsewhere, (see, for example, Ref 8). They are presented here in a convenient vector differential form and an outline of their derivation is included in order to introduce the underlying assumptions.

Wet steam is assumed to be a homogeneous mixture of vapour, at pressure p and temperature T_g , and

spherical water droplets of various sizes. The continuous distributions of droplets is discretized into a number of groups such that the i th group contains n_i droplets per unit mass of mixture of radius r_i and mass m_i . The wetness fraction y_i is then the sum of contributions from all groups and is given by:

$$y = \sum y_i = \sum n_i m_i \quad (1)$$

If the vapour density is ρ_g , the mixture density (neglecting the volume of the liquid phase) is:

$$\rho = \frac{\rho_g}{1 - y} \quad (2)$$

and the mixture specific enthalpy is:

$$h = (1 - y)h_g + \sum y_i h_i \quad (3)$$

where h_g and h_i are the specific enthalpies of the vapour phase and i th group of droplets respectively.

Considerable mathematical simplification results if it is assumed that there is negligible velocity slip between the phases. Under this condition, and in the absence of nucleation, the n_i remain constant along a streamline even in the general case of three-dimensional flow. The approximation can be justified for the size range of fog droplets found in low pressure nozzles and turbines ($0.05 \mu\text{m} < r < 0.5 \mu\text{m}$), but needs further investigation for high pressure calculations where the droplet sizes tend to be larger ($r < 2.0 \mu\text{m}$).

Adopting this assumption for the present, the gas dynamic equations for inviscid adiabatic unsteady two-phase flow can be written as:

$$\text{Continuity} \quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \quad (4)$$

$$\text{Momentum} \quad \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} + \frac{\nabla p}{\rho} = 0 \quad (5)$$

$$\text{Energy} \quad \frac{\partial}{\partial t} \left[\rho \left(e + \frac{V^2}{2} \right) \right] + \nabla \cdot \left[\rho \mathbf{V} \left(h + \frac{V^2}{2} \right) \right] = 0 \quad (6)$$

Notation

c	Specific heat capacity
e	Specific internal energy
F	Function defined by Eq (24)
H	Stagnation specific enthalpy
h	Specific enthalpy
h_{ig}	Latent enthalpy
Kn	Knudsen number
l	Mean free path
m	Mass of a liquid droplet
n	Number of droplets per unit mass of steam
p	Pressure
\dot{p}	Rate of expansion, $D(\ln p)/Dt$
r	Radius of droplet
s	Specific entropy
T	Temperature
ΔT	Vapour supercooling
ΔT_{cap}	Capillary supercooling

ΔT_x	Excess supercooling, $\Delta T - \Delta T_{cap}$
t	Time
\mathbf{V}	Velocity
y	Wetness Fraction
α	Coefficient of expansion
β	Isothermal compressibility
ξ_T	Thermal energy loss coefficient
ρ	Density
σ	Surface Tension
τ	Thermal relaxation time
ϕ	Mass Flow coefficient

Subscripts

g	Vapour phase
i	Liquid phase (i th droplet group)
s	Saturation
eq	Equilibrium
Lack of a subscript indicates that the variable is a mixture property	

where V is the common velocity of the two phases and e is the specific internal energy of the mixture.

Eqs (4)–(6) are identical to those describing the adiabatic flow of an inviscid single-phase fluid. The differences are apparent, however, when it is recalled that the wetness fraction y in Eqs (2) and (3) is not necessarily the equilibrium value and that h_g and the h_i in Eq (3) are evaluated at temperatures T_g and T_i which are not necessarily equal to the local saturation value T_s .

By combining Eqs (4) and (6), it follows immediately that in steady adiabatic flow it is possible to define a stagnation enthalpy:

$$H = (1 - y)h_g + \sum y_i h_i + \frac{V^2}{2} \quad (7)$$

which remains constant along a streamline. The same is not true, however, if velocity differences exist between the phases.

Steam properties

It is customary to assume that the partial pressure of the droplets is negligible. The equation of state of the vapour phase can then be expressed quite generally by:

$$\rho_g = \rho_g(p, T_g) \quad (8)$$

or:

$$\frac{d\rho_g}{\rho_g} = (\beta p) \frac{dp}{p} - (\alpha T_g) \frac{dT_g}{T_g} \quad (9)$$

where α is the coefficient of thermal expansion and β is the isothermal compressibility. Any convenient equation of state can be employed: that used for the calculations presented below is of truncated virial form⁴.

The density, specific enthalpy, specific entropy and surface tension of the liquid phase are assumed to be functions of temperature only and are given empirically by suitable polynomials. The same is true of the saturation properties and the dynamic viscosity and thermal conductivity of the vapour phase.

Growth of the liquid phase

The growth rate of the liquid phase can be related to the growth rate of individual droplets through Eq (1). Noting that the mass of a droplet of the i th group is given by:

$$m_i = \frac{4}{3}\pi r_i^3 \rho_i \quad (10)$$

it follows that:

$$\frac{Dy}{Dt} = \sum \frac{Dy_i}{Dt} = \sum \frac{3y_i}{r_i} \frac{Dr_i}{Dt} \quad (11)$$

where the differentiation with respect to time is the substantive derivative, ie $D/Dt = \partial/\partial t + V \cdot \nabla$.

Condensation on a liquid droplet is a process which proceeds at a rate governed by the ability of the vapour to conduct the released latent enthalpy away from the droplet surface. A derivation and discussion of the droplet growth equation has been pres-

ented elsewhere⁴ and the generally accepted form, valid over a wide range of pressures and flow regimes, is that of Gyarmathy⁸. This is conveniently expressed as:

$$(h_g - h_i) \frac{Dr_i}{Dt} = \frac{\lambda_g}{r_i \rho_i (1 + 3.78 Kn_i / Pr_g)} (T_i - T_g) \quad (12)$$

where λ_g and Pr_g are the thermal conductivity and Prandtl number of the vapour phase respectively and Kn_i is the Knudsen number of droplets in the i th group given by

$$Kn_i = \frac{\bar{l}_g}{2r_i} \quad (13)$$

\bar{l}_g being the mean free path of a vapour molecule.

By combining a mass transfer equation with Eq (12), Gyarmathy also showed that, to a good approximation, the phase temperature difference is given by:

$$T_i - T_g = \Delta T - \Delta T_{cap,i} \quad (14)$$

where $\Delta T = T_s - T_g$ is the vapour supercooling and $\Delta T_{cap,i}$ is the capillary supercooling of the droplet given by

$$\Delta T_{cap,i} = \frac{2\sigma_i T_s}{\rho_i r_i h_{i,g}} \quad (15)$$

σ_i being the surface tension of the liquid at temperature T_i .

For reasons which will emerge later, it is convenient to define a *thermal relaxation time* by:

$$\tau_{T,i} = \frac{(1 - y)c_{pg} r_i^2 \rho_i (1 + 3.78 Kn_i / Pr_g)}{3\lambda_g y_i} \quad (16)$$

where c_{pg} is the isobaric specific heat capacity of the vapour.

Substituting Eqs (12), (14) and (16) into Eq. (11) gives:

$$(h_g - h_i) \frac{Dy_i}{Dt} = \frac{(1 - y)c_{pg}(\Delta T - \Delta T_{cap,i})}{\tau_{T,i}} \quad (17)$$

Eq (17) shows that the time rate of change of y_i following a given fluid particle is directly proportional to the excess of the vapour supercooling above the liquid capillary supercooling and is inversely proportional to the thermal relaxation time.

Fig 1 shows the thermal relaxation time as a function of droplet radius for various pressures and demonstrates that it varies from about 1 μ s to several ms. The flow transit time through a nozzle or blade passage depends on the rate of expansion, but typically is of the order of 0.1–1.0 ms. It follows that under certain flow conditions thermal relaxation effects can be significant.

Thermal non-equilibrium

The secret of obtaining quantitative information about wet steam flows is to obtain the variation of supercooling throughout the expansion. Once this is known, all other variables follow immediately. Normally the complete system of gas dynamic and droplet growth equations are solved numerically, but this can

be a time consuming business, especially for a poly-dispersed flow with a large number of droplet groups. A much simpler, semi-analytical procedure is, however, possible and this is now described.

Taking the scalar product of the momentum Eq (5) with V and combining the Eq (4) and (6) in the usual way gives:

$$\frac{Dh}{Dt} = \frac{1}{\rho} \frac{Dp}{Dt} \quad (18)$$

(Note that this does not imply zero entropy increase as it would do in single phase flow). Introducing Eq (3) and the growth law (Eq (17)) gives:

$$\begin{aligned} (1-y) \frac{Dh_g}{Dt} + \sum y_i \frac{Dh_i}{Dt} - \sum \frac{(1-y)c_{pg}(\Delta T - \Delta T_{cap,i})}{\tau_{T,i}} \\ = \frac{(1-y) Dp}{\rho_g Dt} \end{aligned} \quad (19)$$

The change in vapour enthalpy can be expressed using standard thermodynamic relationships by:

$$\frac{Dh_g}{Dt} = c_{pg} \frac{DT_g}{Dt} + \frac{(1-\alpha T_g) Dp}{\rho_g Dt} \quad (20)$$

The changes in the h_i are given by:

$$\frac{Dh_i}{Dt} = c_i \frac{DT_i}{Dt} \approx c_i \frac{DT_s}{Dt} \quad (21)$$

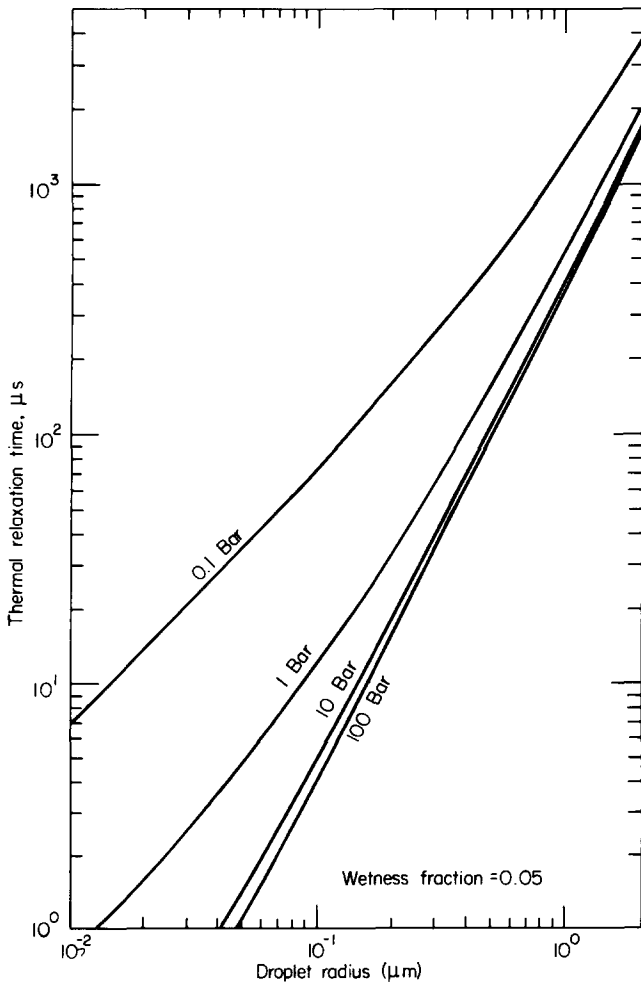


Fig 1 Thermal relaxation time

where c_i is the specific heat capacity of the liquid and where the change in capillary supercooling of each droplet group has been ignored, this being quite negligible.

Substituting Eqs (20) and (21) in Eq (19) and using the Clausius-Clapeyron equation in the form:

$$\frac{DT_s}{Dt} = \frac{T_s}{\rho_s h_{fg}} \frac{Dp}{Dt} \quad (22)$$

gives, after some manipulation:

$$\frac{D}{Dt}(\Delta T) + \sum \frac{(\Delta T - \Delta T_{cap,i})}{\tau_{T,i}} = F \left(\frac{1}{p} \frac{Dp}{Dt} \right) \quad (23)$$

where:

$$F = \frac{p}{(1-y)c_{pg}\rho_s} \left[\frac{cT_s}{h_{fg}} - (1-y)(\alpha T_g) \frac{\rho_s}{\rho_g} \right] \quad (24)$$

c being the specific heat capacity of the mixture:

$$c = (1-y)c_{pg} + \sum y_i c_i \quad (25)$$

Eq (23) can be written:

$$\frac{D}{Dt}(\Delta T) + \frac{(\Delta T - \Delta T_{cap})}{\tau_T} = F \frac{D(\ln p)}{Dt} \quad (26)$$

where:

$$\frac{1}{\tau_T} = \sum \frac{1}{\tau_{T,i}} \quad (27)$$

and:

$$\Delta T_{cap} = \sum \frac{\tau_T}{\tau_{T,i}} \Delta T_{cap,i} \quad (28)$$

This demonstrates how thermal non-equilibrium effects in a polydispersed flow of wet steam may be represented computationally by an equivalent monodispersed flow. Eqs (27) and (28) define suitable average values of thermal relaxation time and capillary supercooling (and hence an average droplet radius) which, when used in conjunction with Eq (26) predict the same value of supercooling as is observed with a fully polydispersed flow.

Defining the *excess supercooling* ΔT_x by:

$$\Delta T_x = \Delta T - \Delta T_{cap} \quad (29)$$

and neglecting the variation of ΔT_{cap} with time, Eq (26) becomes:

$$\frac{D}{Dt}(\Delta T_x) + \frac{\Delta T_x}{\tau_T} = F \dot{p} \quad (30)$$

where Gyarmathy's notation of \dot{p} has been used for the rate of expansion $D(\ln p)/Dt$. ΔT_x represents the driving potential for phase change or, alternatively, a measure of the departure from thermal equilibrium. Except for very small droplets it is substantially the same as ΔT , ($\Delta T_{cap} \approx 1K$ for $r \approx 0.02 \mu m$), and the difference will often be ignored in the remaining analysis.

Apart from very minor approximations, Eq (30) is an exact first order differential equation for the variation with time of supercooling of a fluid particle and is valid for the general case of unsteady, three-dimensional flow. It has not appeared in the literature

previously and is an extremely convenient formulation for all types of wet steam calculation. F is a function predominantly of the saturation conditions and its variation with pressure, which is very slight, is shown in Fig 2 for zero wetness fraction and supercooling.

For values of τ_T which are small compared with the flow transit time, Eq (30) is the archetype of a mathematically *stiff* differential equation. In these circumstances numerical integration procedures using conventional finite differencing would lead to the unstable amplification of errors unless very small time increments were employed. Most computational problems associated with the numerical solution of the wet steam equations can be traced to this cause. The solution is to integrate Eq (30) analytically over time increments such that τ_T and $F\dot{p}$ remain sensibly constant. As τ_T changes only slowly through an expansion, the size of the increments is then dictated by the rate of change of flow properties rather than by the relaxation time. Adopting this technique, Eq (30) can be integrated to give:

$$\Delta T_x = \Delta T_{x0} e^{-t/\tau_T} + \tau_T F \dot{p} (1 - e^{-t/\tau_T}) \quad (31)$$

where ΔT_{x0} is the excess supercooling at the start of the time increment, $t=0$. Eq (31) can be applied stepwise through the expansion, the accuracy obtained being controlled by the size of the increments used. Usually very large steps can be taken and need only be reduced in regions where the expansion rate changes rapidly. (In practice quite reasonable estimates of the departure from equilibrium may be obtained easily by approximating the expansion with a single step and adopting average values of τ_T , F and \dot{p} .)

When dealing with polydispersed flows, a word of caution is in order. The validity of the use of a single relaxation time to characterise a polydispersion remains to be established. It is therefore possible that under certain circumstances the relaxation times for individual droplet size groups may change quite rapidly, for example following a nucleation, thus making the integration of Eq (30) by the above method prone to error.

The variation of wetness fraction throughout the expansion can be obtained by combining Eq (1), (17), (27), (28) and (29) to give:

$$\frac{Dy}{Dt} = \frac{(1-y)c_{pg}\Delta T_x}{h_{fg}\tau_T} \quad (32)$$

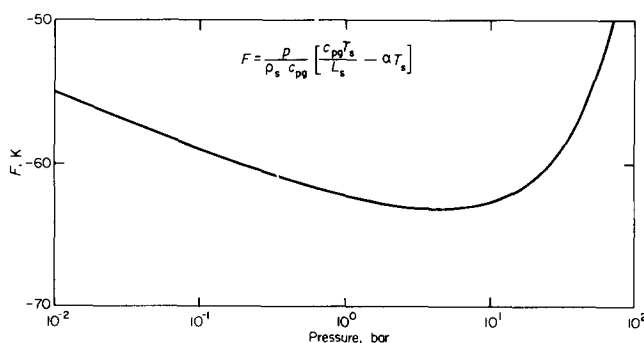


Fig 2 Variation of F with pressure

where the terms $(h_g - h_i)$ have been approximated by h_{fg} , the latent enthalpy. Substituting Eq (31) and integrating gives:

$$y - y_0 = \frac{(1-y)c_{pg}}{h_{fg}} [(\Delta T_{x0} - \Delta T_x) + F\dot{p}t] \quad (33)$$

where y_0 is the wetness fraction at the start of the increment and a suitable average value is assumed for the slowly varying function $(1-y)c_{pg}/h_{fg}$. Eq (33) is also only valid over increments such that $F\dot{p}$ is sensibly constant.

It is now possible to demonstrate in a particularly simple way many of the effects of thermal non-equilibrium. For clarity, the discussion is limited to flows where τ_T and $F\dot{p}$ remain constant, but broadly similar conclusions apply to cases where the rate of expansion is continually changing.

First, it is evident from Eqs (31) and (33) that two limiting cases can be identified. For very small droplets $\tau_T \rightarrow 0$ and:

$$\left. \begin{aligned} \Delta T_x &\rightarrow 0 \\ y &\rightarrow y_{eq} = y_0 + \frac{(1-y)c_{pg}}{h_{fg}} (F\dot{p}t) \end{aligned} \right\} \quad (34)$$

corresponding to *equilibrium flow*. Conversely the limiting case of *frozen flow* occurs for large droplets where $\tau_T \rightarrow \infty$ and:

$$\left. \begin{aligned} \Delta T_x &\rightarrow \Delta T_{x0} + F\dot{p}t \\ y &\rightarrow y_0 \end{aligned} \right\} \quad (35)$$

For intermediate values of τ_T , the excess supercooling approaches a constant value as t becomes large.

$$\Delta T_x \rightarrow \tau_T F \dot{p} \quad \text{for } t \gg \tau_T \quad (36)$$

Somewhat surprisingly, therefore, the supercooling does not increase indefinitely, but tends to a steady state condition of dynamic equilibrium governed by the product of τ_T and \dot{p} . As Eq (31) clearly shows, however, the duration of the transient phase leading to this condition is determined solely by the thermal relaxation time and not by the expansion rate.

Combining Eqs (33) and (34) yields:

$$(y_{eq} - y) = \frac{(1-y)c_{pg}}{h_{fg}} (\Delta T_x - \Delta T_{x0}) \quad (37)$$

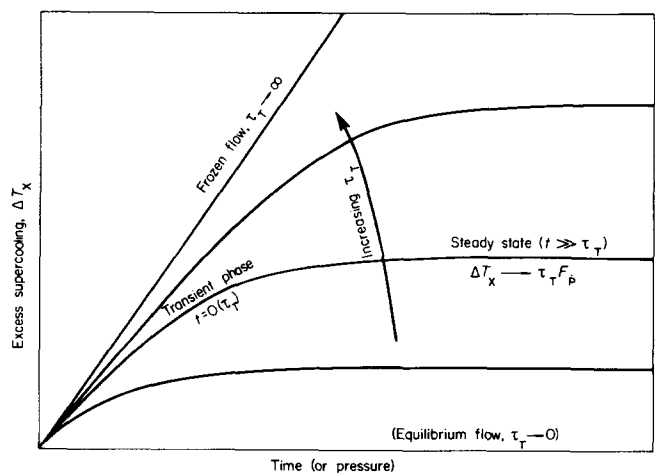


Fig 3 Development of supercooling in an expansion

showing that the deviation from the equilibrium wetness fraction is a function only of ΔT_x and therefore tends also to a constant value after the initial transient phase.

All the effects described above are shown schematically in Figs 3 and 4 which show the development of the excess supercooling and the wetness fraction as functions of time for various values of τ_T .

Thermal losses

The temperature difference between the liquid and vapour phases causes an irreversible heat and mass transfer resulting in an increase in entropy of the mixture. This process is usually referred to as the *thermodynamic loss* and is partially responsible for the reduction in work output of turbines operating below the saturation line.

It has been shown by a number of authors that the rate of increase of entropy due to thermal non-equilibrium effects (Ds_T/Dt) is given, to a good approximation, by:

$$\frac{Ds_T}{Dt} \approx \frac{h_{fg}\Delta T}{T_s^2} \frac{Dy}{Dt} \approx \frac{(1-y)c_{pg}}{T_s^2} \frac{\Delta T^2}{\tau_T} \tag{38}$$

Eq (38) is normally solved numerically, but can, in fact, be integrated analytically over regions of constant $F\dot{p}$ by introducing the supercooling equation (Eq (31)). Neglecting ΔT_{cap} the result is:

$$\Delta s_T = \frac{(1-y)c_{pg}}{T_s^2} \left\{ \frac{\Delta T_0^2}{2} (1 - e^{-2t/\tau_T}) + F\dot{p}\tau_T \Delta T_0 (1 - e^{-t/\tau_T})^2 + (F\dot{p}\tau_T)^2 \left[\frac{t}{\tau_T} - 2(1 - e^{-t/\tau_T}) + \frac{1}{2}(1 - e^{-2t/\tau_T}) \right] \right\} \tag{39}$$

Consider now an expansion from equilibrium ($\Delta T_0 = 0$) with a constant rate of expansion. According to Eq (39) the entropy increase is given by:

$$\Delta s_{TE} = \frac{(1-y)c_{pg}}{T_s^2} (F\dot{p}\tau_T)^2 \left[\frac{t}{\tau_T} - 2(1 - e^{-t/\tau_T}) + \frac{1}{2}(1 - e^{-2t/\tau_T}) \right] \tag{40}$$

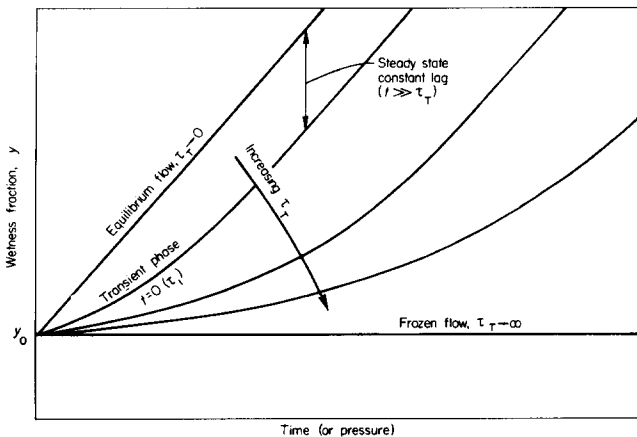


Fig 4 Growth of the liquid phase in an expansion

For both equilibrium flow ($\tau_T \rightarrow 0$) and frozen flow ($\tau_T \rightarrow \infty$), Eq (40) shows that $\Delta s_{TE} \rightarrow 0$. Intermediate values of τ_T , however, give positive increases in entropy.

This surprising result suggests that the thermodynamic loss increases with τ_T , passes through a maximum and decreases to zero for large values of τ_T . In arriving at this conclusion, however, the full implications of the departure from equilibrium have been neglected.

At each stage of an expansion the deviation from thermal equilibrium is measured by the supercooling. If the expansion were terminated at this point a relaxation process would subsequently occur in which the supercooling decayed to zero. If this were to happen at constant pressure (an excellent assumption for the flow emerging from a blade passage) Eq (39) shows that the corresponding entropy increase would be:

$$\Delta s_{TR} = \frac{(1-y)c_{pg}}{T_s^2} \frac{\Delta T^2}{2} \tag{41}$$

ΔT being the supercooling at the end of the expansion or start of the relaxation zone. In any real process it is impossible to avoid this relaxation loss and hence the total thermodynamic loss at any stage in the expansion is given more realistically by:

$$\Delta s_T = \Delta s_{TE} + \Delta s_{TR} \tag{42}$$

Hence

$$\Delta s_T = \frac{(1-y)c_{pg}}{T_s^2} (F\dot{p}\tau_T)^2 \left[\frac{t}{\tau_T} - (1 - e^{-t/\tau_T}) \right] \tag{43}$$

Equation (43) shows that the total thermodynamic loss increases monotonically with thermal relaxation time.

The increase in entropy in a non-equilibrium expansion causes a reduction in efficiency of nozzles or turbine blades because the exit velocity is lower than the corresponding exit velocity when the flow is isentropic. It is interesting to note, however, that from the point of view of the nozzle performance, the total loss Δs_T given by Eq (43) has effectively been incurred at the end of the expansion (before the relaxation process) where the entropy increase is given by Δs_{TE} . The explanation can be found in Fig 5 which

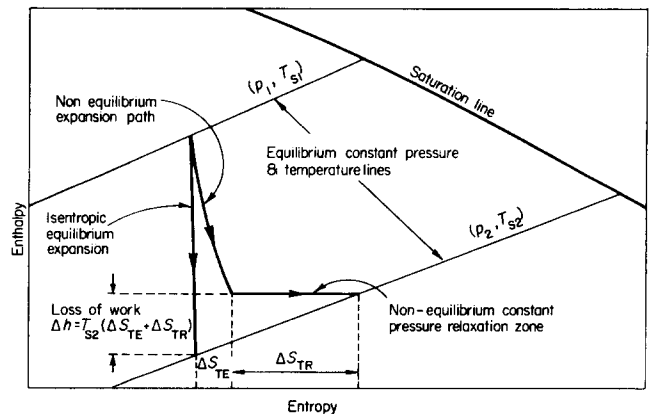


Fig 5 Entropy production in a non-equilibrium expansion

shows the state path for a typical expansion on a non-equilibrium enthalpy-entropy diagram. During the constant pressure relaxation process the enthalpy, and hence the steam velocity, remains constant although the entropy increases. Thus the relaxation process simply involves a heat and mass transfer between the phases in order to equalise the temperatures, the velocity remaining constant. It is, therefore, entirely appropriate to represent the thermodynamic loss at any stage in an expansion by Eq (43) despite the fact that, at that point, the only entropy increase incurred is that given by Eq (40). Conceptually it is possible for the relaxation process to take place with zero increase in entropy for one could imagine the temperature difference between a droplet and the surrounding vapour being used to drive a reversible heat engine, the work output from which could be used to accelerate the gas stream. Practically, of course, this is totally impossible.

From an engineering viewpoint, it is useful to represent the thermodynamic entropy increase in the form of a loss coefficient. The turbine designer has access to a large number of correlations for profile and secondary viscous losses, but at present has little idea of the magnitude of the thermodynamic loss. The most suitable form is the *thermal energy loss coefficient* defined by:

$$\xi_T = \frac{T_s \Delta s_T}{(V^2/2)} \quad (44)$$

where Δs_T is the total thermodynamic entropy increase, V is the nozzle or blade exit relative velocity and T_s is the saturation temperature at the exit static pressure.

Applications

The theory described in the previous sections can be used in conjunction with a number of different

turbomachinery calculation procedures to investigate non-equilibrium effects in turbines. In order to apply Eqs (31), (33) and (43), however, it is necessary to know, at least approximately, the pressure distribution throughout the flow field. Experience with numerical wet steam calculations has indicated that there is usually very little difference between the pressures calculated under equilibrium or non-equilibrium conditions. An excellent approximation can therefore be obtained by adopting the equilibrium distribution, which is usually available anyway as the result of pure aerodynamic calculations. More accurate results can be generated by iterating on the pressure, but this is hardly necessary and is not in the spirit of the present work, which seeks to provide simple methods of reasonable accuracy for estimating the importance of non-equilibrium phenomena.

Thermodynamic losses in nozzles

The first example concerns the effect of rate of expansion and droplet size on the thermodynamic loss coefficient in nominally one-dimensional nozzles. Normally the nozzle cross-sectional area distribution is specified and this can be used to generate the equilibrium pressure distribution for use in Eqs (31), (33) and (43). For the purposes of illustration, however, calculations were performed with nozzles having constant rates of expansion. The flow was assumed to be steady and hence the time to traverse a section of nozzle was uniquely related to the corresponding pressure drop. The inlet stagnation pressure and wetness fraction were arbitrarily taken to be 1 bar and 0.05 respectively and the steam was assumed to be initially at equilibrium.

By applying Eqs (31), (33) and (43), the exit supercooling, wetness fraction and thermal energy loss coefficient were calculated directly for various nozzle pressure ratios. Figs 6 and 7 show the results

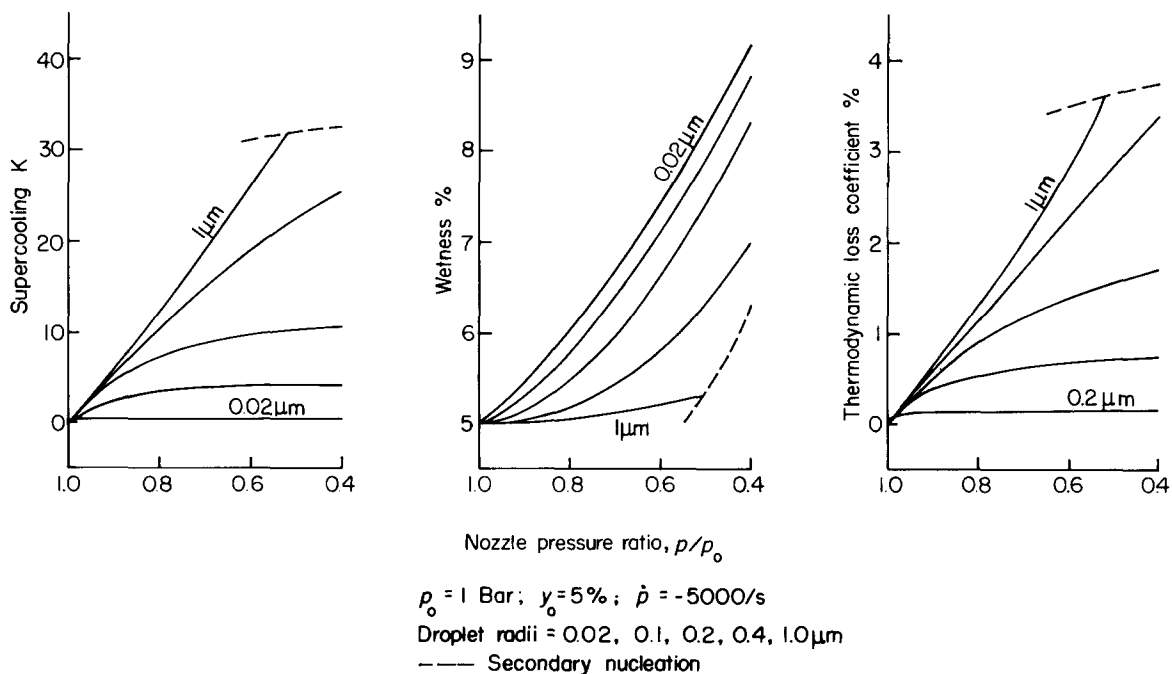


Fig 6 Variation of ΔT , y and ξ_T in an expansion (constant expansion rate, various droplet sizes)

of the calculations for a range of droplet sizes and rates of expansion. It is evident that, under certain conditions, the thermodynamic losses in a wet steam turbine stage can be of comparable magnitude to the profile and secondary losses.

If the supercooling rises above a certain level a secondary nucleation will occur and the analysis is no longer valid. For a given pressure and supercooling, the nucleation rate can be calculated from the standard non-isothermal classical theory recommended by a number of authors. Calculations indicate that if the rate exceeds a nominal value of about $10^{18} \text{ m}^{-3} \text{ s}^{-1}$ a rapid increase in the number of droplets is imminent. This threshold is indicated in Figs 6 and 7.

Mass flow coefficient

It is known that the mass flowrate passed by a wet steam nozzle or blade passage operating at a specified pressure ratio depends on the supercooling developed during the expansion. A *mass flow coefficient* ϕ can therefore be defined such that:

$$\phi = \frac{\text{non-equilibrium mass flowrate}}{\text{equilibrium mass flowrate}} \quad (45)$$

both values being evaluated at the same pressure ratio.

Calculations of ϕ can be performed simply. Having obtained the exit supercooling and wetness fraction as described previously, the flow velocity can be obtained from the energy equation (Eq (7)) and the vapour density from the equation of state (Eq (8)). The mass flowrate per unit area can then be calculated and compared with the corresponding equilibrium value.

Fig 8 shows computed values of ϕ as a function of nozzle pressure ratio for various constant expansion rates and droplet sizes. It can be seen that the actual mass flowrate can be several per cent higher than the

value calculated from equilibrium theory. Note that the calculations were terminated at a pressure ratio such that the exit velocity was close to, but not quite, sonic. The problem of choking in wet steam flows is complex and will be dealt with in a future paper.

Blade-to-blade flows

In many cases a one-dimensional calculation will be deemed insufficient and it may be required to investigate circumferential variations in supercooling and wetness fraction, etc. Eqs (31), (33) and (43) can still be applied, however, as they are also valid along streamlines even in the general case of steady three-dimensional flow.

An approximation to the pressure distribution in a two-dimensional blade-to-blade plane can be generated using any available equilibrium computational method. For example, the pressure distribution around a typical steam turbine blade profile is shown in Fig 9 and was obtained using Denton's time-marching method⁹. Eqs (31) and (33) were then applied to obtain the distributions of supercooling and wetness fraction around the pressure and suction surfaces of the blade for various assumed droplet sizes. The results are also shown in Fig. 9 and it is evident that significant variations can exist in the pitchwise direction. Furthermore, secondary nucleations may occur, originating on the suction surface and extending either part or whole way across the blade passage. Note that a more accurate estimate of the thermodynamic loss coefficient can also be obtained by applying Eq (43) along a number of streamlines and mass averaging the results.

Hub-to-tip flows

Throughflow analysis of large low pressure turbines using streamline curvature or matrix methods is now

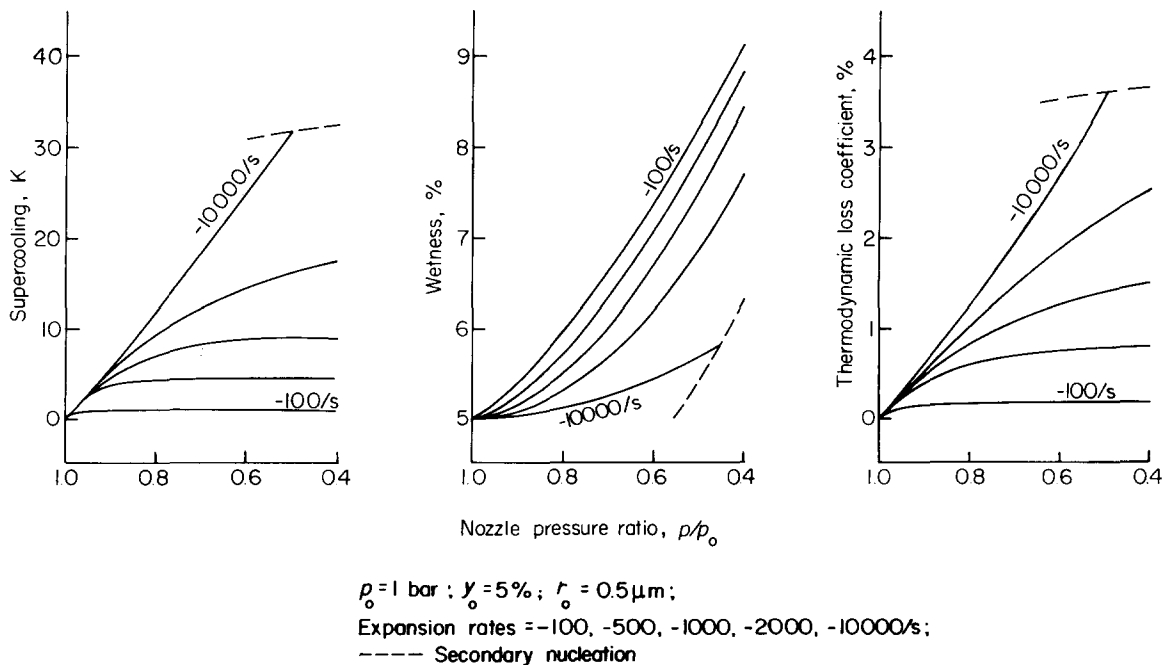


Fig 7 Variation of ΔT , y and ξ_T in an expansion (constant droplet radius, various expansion rates)

an established design technique used by all manufacturers. With the exception of one method⁶, however, the steam is assumed to be in equilibrium below the saturation line. User experience of the computer program given by Yeoh and Young⁶ suggests that changes in the calculated pressure field due to the inclusion of the non-equilibrium terms are slight. Good estimates of the deviation from equilibrium can therefore be obtained by applying the theory given here to pressure distributions calculated from an equilibrium program. The savings in computational time and the gain in physical understanding are considerable. Obviously the results are subject to the limitations imposed by the axisymmetric assumption, but circumferential variations can always be investigated using the approach described in the last section.

Fig 10 shows the spanwise pressure distribution at three stations in the last stage of a modern low

pressure steam turbine used for electricity generation. The results were obtained using Denton's equilibrium streamline curvature throughflow program¹⁰. Using this approximate pressure distribution as a basis, the deviations from equilibrium were calculated for a range of droplet sizes. Fig 10 shows the spanwise distribution of the thermodynamic loss coefficient at the nozzle and rotor trailing edges. The maximum departures from equilibrium are found at the stator hub and rotor tip where the relative velocities and rates of expansion are highest and these are regions of maximum thermodynamic loss. There is considerable variation of supercooling in the spanwise direction and this can be high enough to trigger a secondary nucleation depending on the size of the droplets.

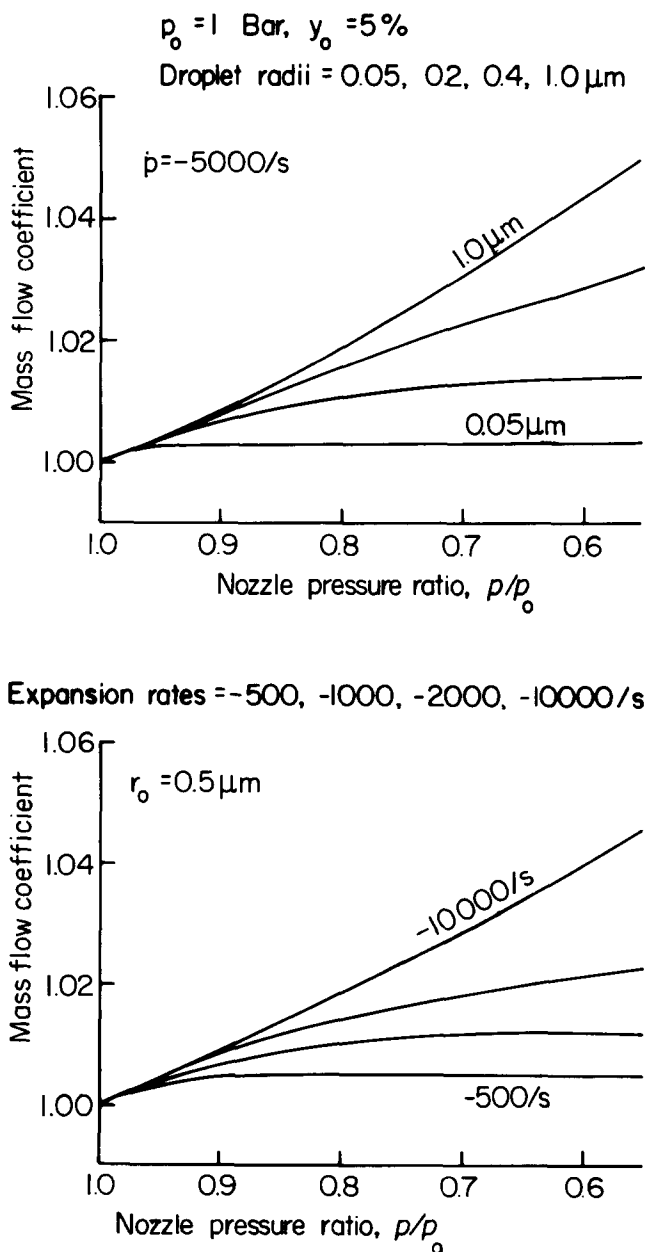


Fig 8 Variation of mass flow coefficient with nozzle pressure ratio

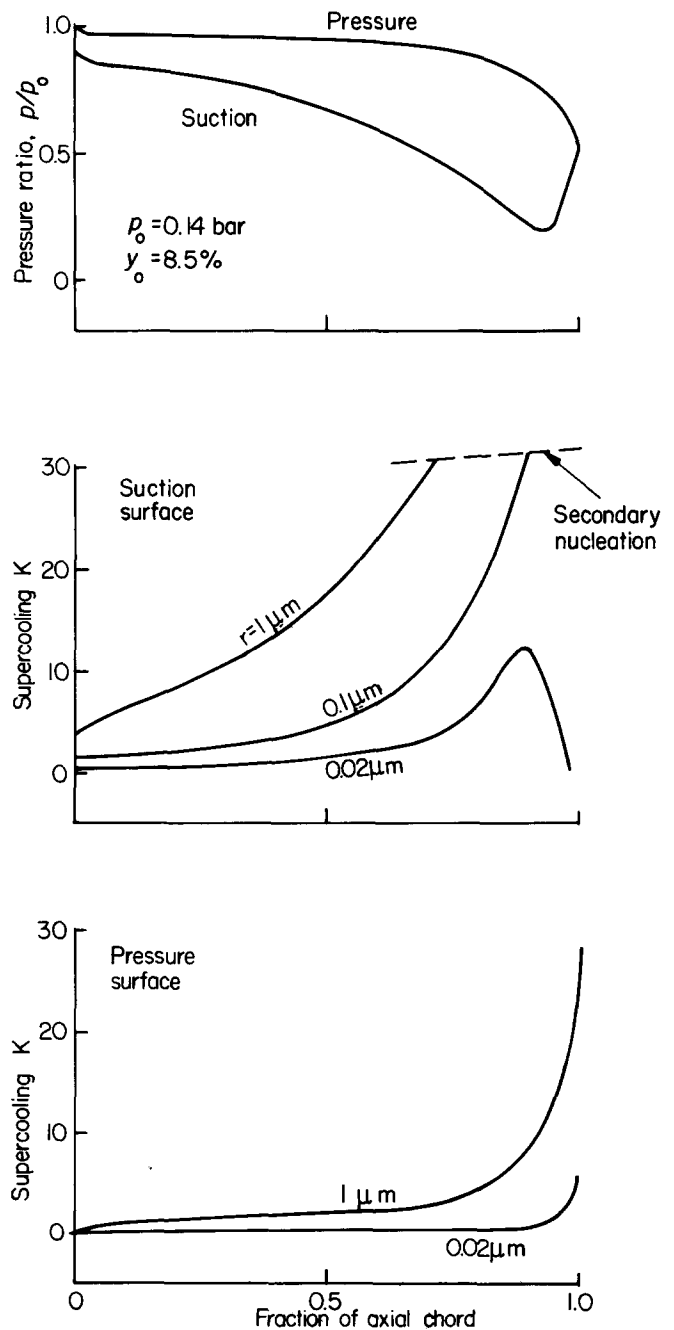


Fig 9 Departures from equilibrium in a two-dimensional steam turbine cascade

Flow incidence variations

Departures from equilibrium in one blade row can be responsible for changes to the inlet flow angle of the next. In general this is the result of two effects. Firstly, the flow exit velocity from row one may be significantly altered due to the supercooling developed during the expansion. Assuming the relaxation zone downstream of the blade to be at constant

pressure, the momentum equation (Eq (5)) shows that there is no change in velocity as the steam reverts to equilibrium. In practice, of course, there may be small changes of static pressure between the blade rows, but the principle still holds that the deviation from the equilibrium velocity at inlet to the second row is closely related to the deviation which existed at the trailing edge of the first. The non-equilibrium velocity is usually lower than the corresponding equilibrium value and construction of the relevant velocity triangle shows that the flow enters the second blade passage with increased negative incidence irrespective of whether it is a stator or a rotor blade.

The second effect involves a turning of the flow downstream of the first blade row. In the relaxation zone, assumed to be at constant pressure and hence constant velocity, the vapour temperature rises as the supercooling is depleted and equilibrium is re-established. This results in a decrease in mixture density, despite an increase in wetness fraction, and a larger cross-sectional area is required to pass the given mass flowrate. The velocity vector therefore turns towards the axial direction as the supercooling falls to zero. As before, the velocity triangle shows that this effect also results in increased negative incidence onto the following row for both stator and rotor blades.

Calculations of the change in flow incidence have been performed for the turbine stage discussed in the previous section. Fig 11 shows the deviation from the equilibrium rotor inlet flow angle. Although the deviations are comparatively small, some deterioration in blade performance might result, especially in the region of the rotor tip where the profile is highly sensitive to small changes in imposed flow angle.

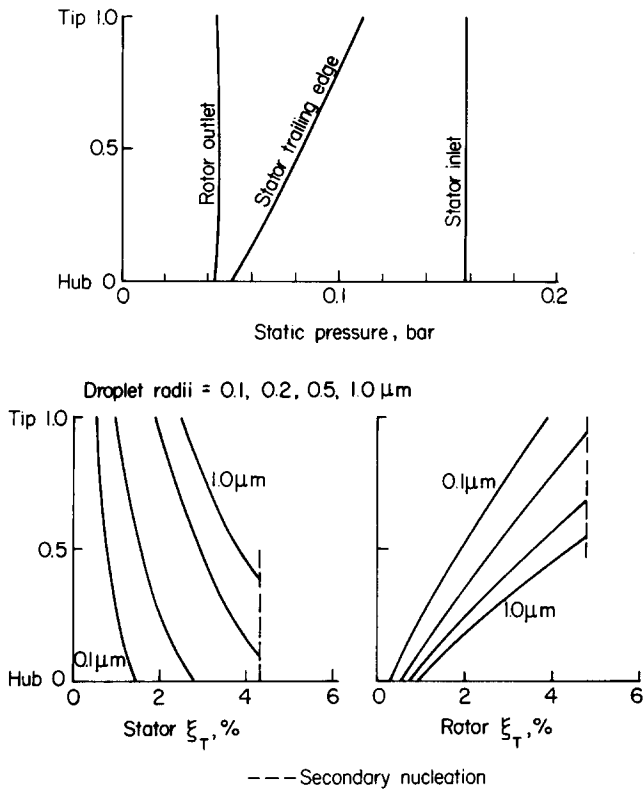


Fig 10 Spanwise variation of thermodynamic losses in the last stage of an LP turbine

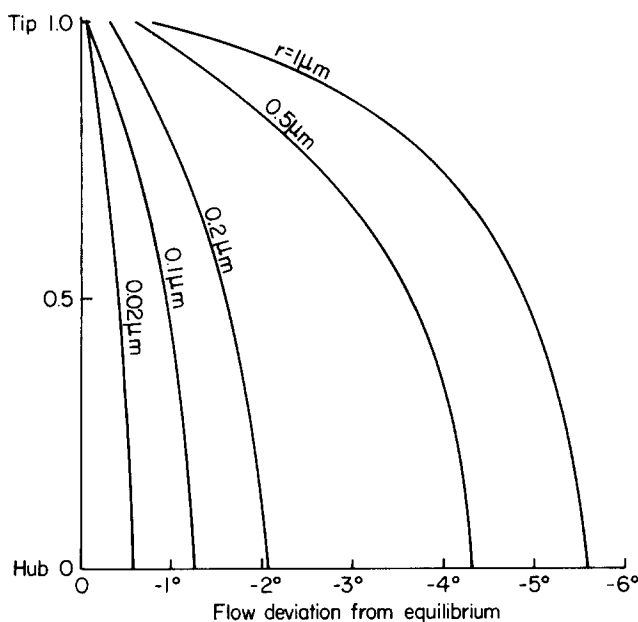


Fig 11 Variation of flow incidence onto a final stage rotor blade

Conclusions

A simple, but versatile, method for estimating non-equilibrium effects in non-nucleating wet steam flows has been described. Apart from requiring an initial estimate of the pressure distribution, the technique is self-consistent and the resulting equations involve only minor approximations. It can be used in conjunction with any conventional turbomachinery calculation procedure to estimate the supercooling distribution, the deviation from the equilibrium wetness fraction, the thermodynamic losses and other flow variables of interest. In order to demonstrate the usefulness of the method, a number of applications in nozzle and turbine flows have been described.

The technique presented is not meant to replace the complete numerical analyses at present being developed by a number of different authors. Rather it is meant to provide a framework for a better understanding of the behaviour of wet steam flows.

All the calculations described could have been obtained numerically, but the beauty of the semi-analytical approach is that it furnishes general results and physical insight at low computational expense and effort. These features are of considerable value in an area which has produced few concrete results of use to the turbine designer despite sixty years of research.

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